A Finite Element Approach to Reinforced Concrete Slab Design in GT STRUDL

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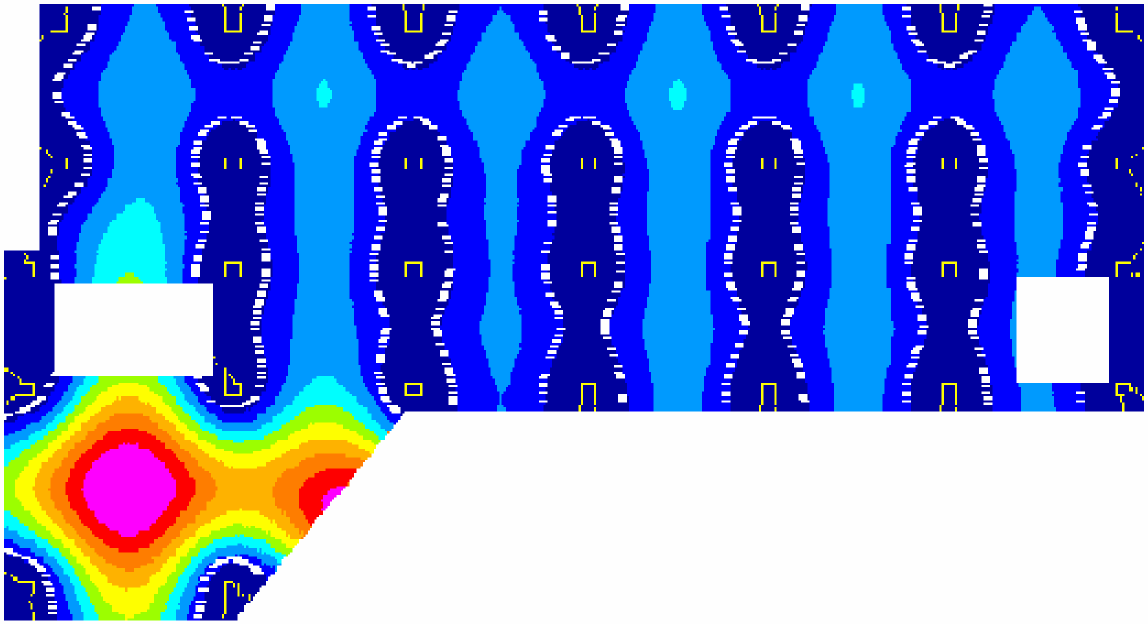
Georgia Institute of Technology

Introduction

- Background and Motivation
- The Wood and Armer Method
- The Element Force Method
- Implementation in GT STRUDL
- Case Studies
- Conclusions and Recommendations

Objective

- Design reinforcing steel for reinforced concrete elevated slabs based on the results of finite element analysis



Background

- Finite Element Analysis has been applied to reinforced concrete systems since 1967
- Finite elements, like concrete, can conform to any shape
  - Slabs
  - Shear Walls
  - Shells
- One of many techniques regularly applied to design of reinforced concrete flat plate slabs and mat foundations
  - Especially irregular or unusual slabs

Motivation

- Limitations of common slab design techniques:
  - Direct Design / Equivalent Frame Technique:
    - Limited Applicability – very regular structures
  - Strip Design
    - Based on assumed location of yield lines
    - Can lead to over-conservative design and/or poor serviceability
  - Classical plate theory
    - Solution of 4^{th} order partial differential equation
- Solution to these limitations: <u>Finite Elements</u>

Motivation

- Finite elements easily handle complexities that restrict simplified design methods
  - Holes, concentrated loads, irregularities
- Primary limiting factors with finite elements:
  - Difficulty in interpretation of results
  - Volume of results
- Demonstrates need for a conservative *tool* to simplify design of reinforced concrete using finite elements

Linear-Elastic Analysis

- Design based on linear-elastic distribution of moments
  - Standard practice in industry
  - Research/practice confirms its applicability
  - ACI Code allows it:
    - "All members of frames or continuous construction shall be designed for the maximum effects of factored loads as determined by the theory of elastic analysis…" (8.3.1)
    - "A slab system shall be designed by any procedure satisfying conditions of equilibrium and geometric compatibility, if shown that the design strength at every section is at least equal to the required strength…" (13.5.1)

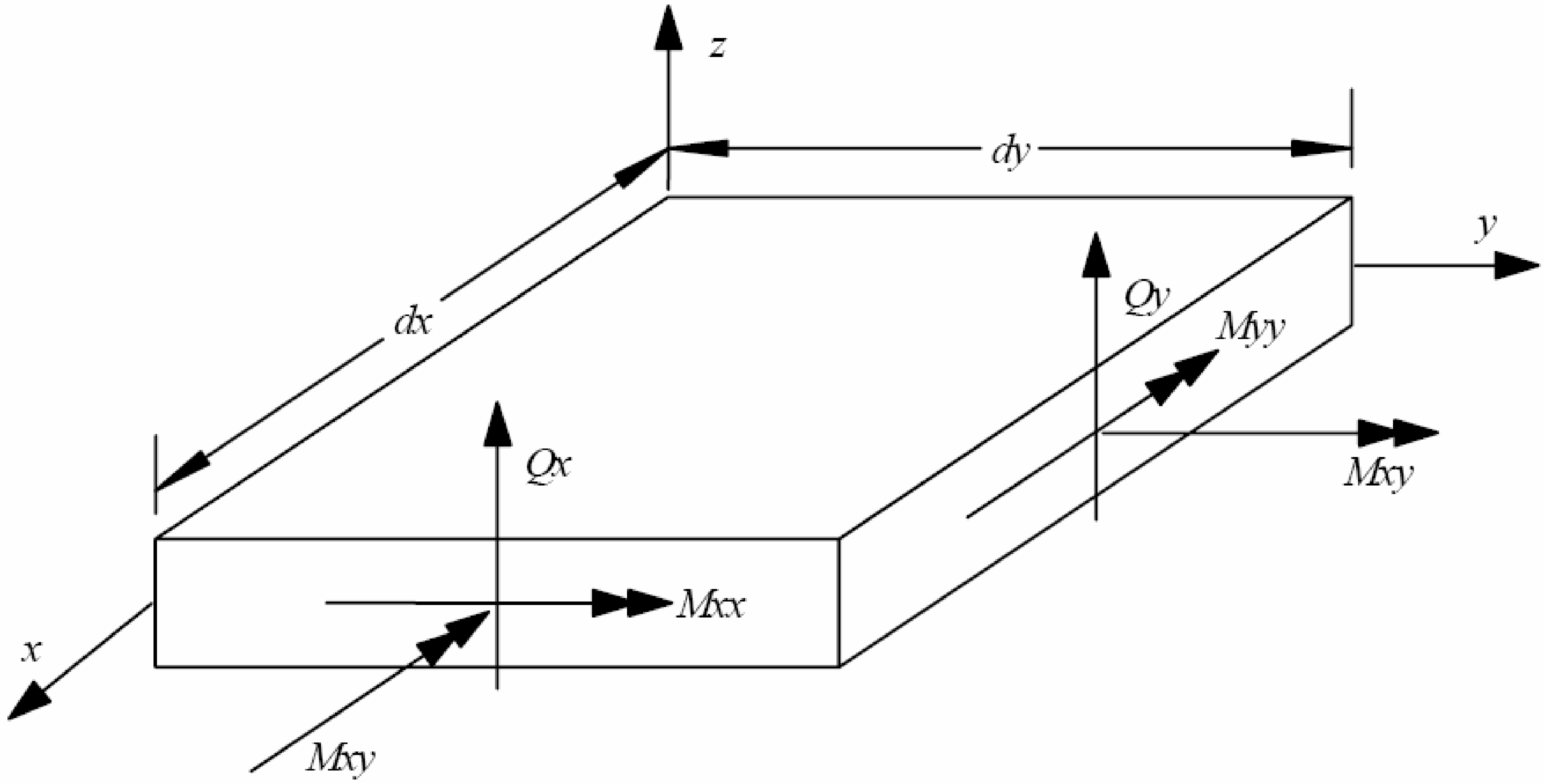
Slab Design Based on the FEM

- 2 General Methods
  1. Moment Resultants (Wood & Armer)
  2. Element Forces (CALCULATE RESULTANT)
- Both methods are fundamentally different with respect to underlying theory and assumptions
- Thus, each method has advantages over the other under certain circumstances.
- Most commercial software packages only offer ONE of these methods.
- Both methods depend on defining a “cut”, comprised of a list of co-linear nodes and adjacent elements

Method 1 – Moment Resultants

- Moment "stresses" computed at each node of each element from strains and constitutive model
- Units: Bending Moment per unit width
  - kip-in/in or kip-ft/ft
- Can be very inaccurate if mesh is not suitable for problem

Moment Resultant Conventions



$$M_{xx} = \int_{-t/2}^{t/2} \sigma_x z \mathrm{d}z \qquad M_{yy} = \int_{-t/2}^{t/2} \sigma_y z \mathrm{d}z \qquad M_{xy} = \int_{-t/2}^{t/2} \tau_{xy} z \mathrm{d}z$$

Computation of Moment Resultants

- Computed per element:
- Strains computed from displacements
- Moment results computed from rigidity

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = - \begin{bmatrix} D & \nu D & 0 \\ \nu D & D & 0 \\ 0 & 0 & \frac{(1-\nu)D}{2} \end{bmatrix} \begin{Bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{Bmatrix}$$

$$D = \frac{Et^3}{12(1-\nu^2)}$$

Moment Resultants in GT STRUDL

- Use CALCULATE AVERAGE command
- Averages results at each node with contributions only from elements in the element list
- Gives M_{xx}, M_{yy}, and M_{xy} in PLANAR system

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CALCULATE AVERAGE RESULTANTS AT
MIDDLE ELEMENTS EXISTING list
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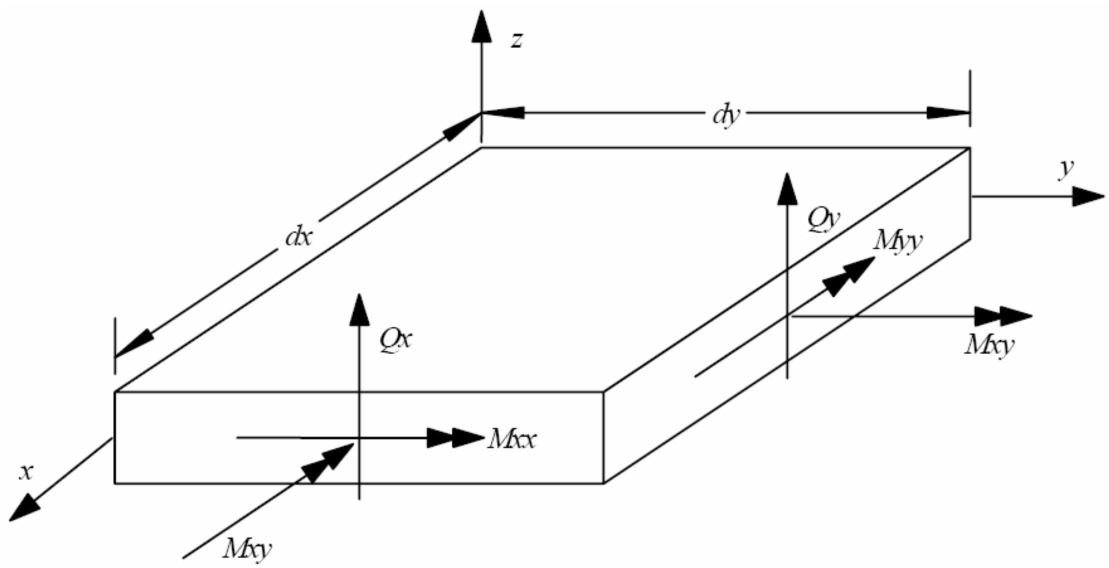
Wood and Armer, 1

- The graphic below shows an area *dA* which effectively is the size of a node



- M_{xy} contributes a moment effect to both the principal bending directions *x* and *y*

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Wood and Armer, 2

- To Design Bottom Reinforcement:

$$M_{ux}^{+} = M_{xx} + |M_{xy}|$$

$$M_{uy}^{+} = M_{yy} + |M_{xy}|$$

$$\text{If} \quad M_{ux}^{+} < 0 \quad \text{then} \quad M_{ux}^{+} = 0 \quad \text{and} \quad M_{uy}^{+} = M_{yy} + \left| \frac{M_{xy}^2}{M_{xx}} \right|$$

$$\text{If} \quad M_{uy}^{+} < 0 \quad \text{then} \quad M_{uy}^{+} = 0 \quad \text{and} \quad M_{ux}^{+} = M_{xx} + \left| \frac{M_{xy}^2}{M_{yy}} \right|$$

Wood and Armer, 3

- To Design Top Reinforcement:

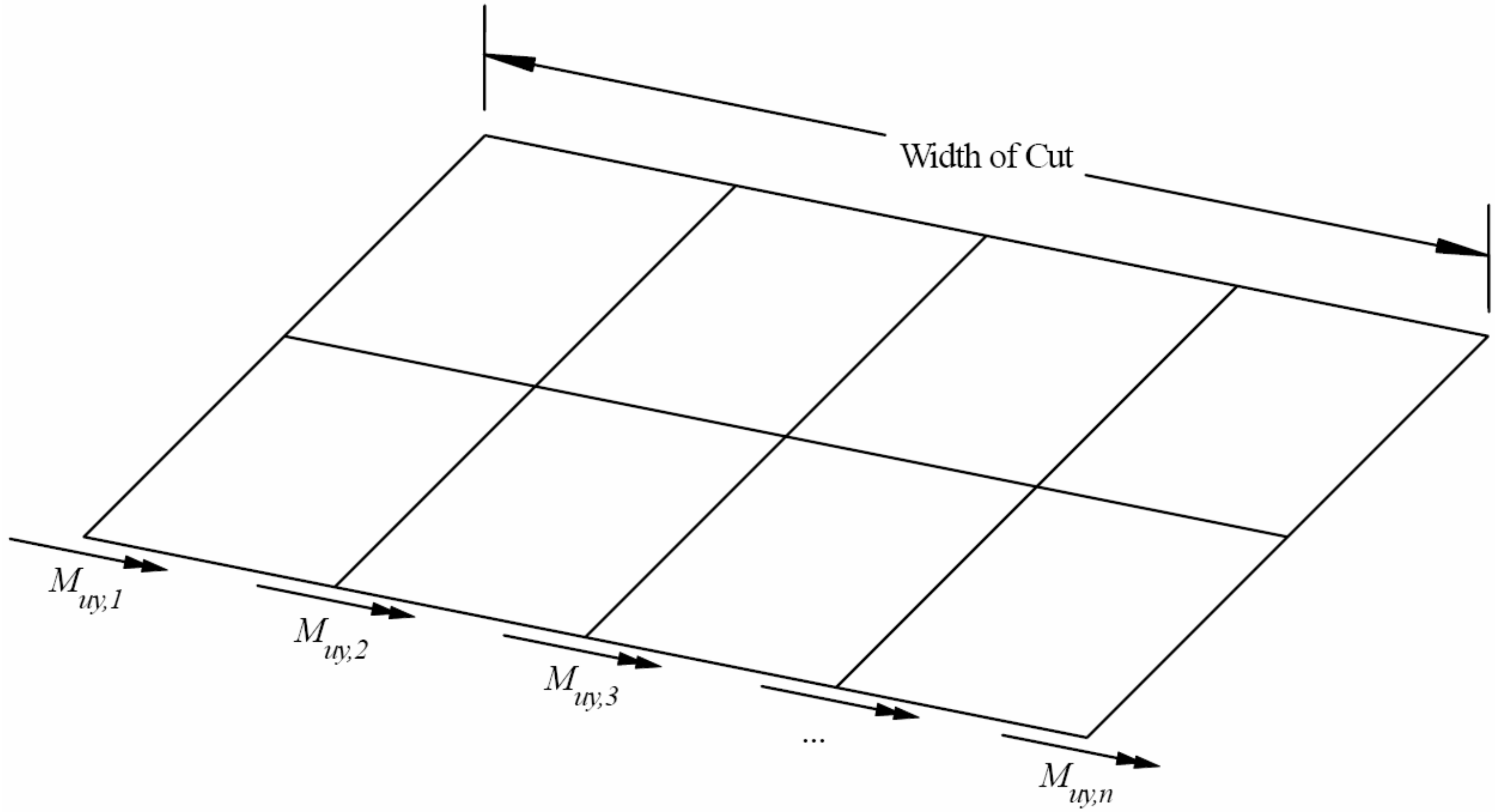
$$M_{ux}^{-} = M_{xx} - |M_{xy}|$$

$$M_{uy}^{-} = M_{yy} - |M_{xy}|$$

$$\text{If} \quad M_{ux}^{-} > 0 \quad \text{then} \quad M_{ux}^{-} = 0 \quad \text{and} \quad M_{uy}^{-} = M_{yy} - \left|\frac{M_{xy}^2}{M_{xx}}\right|$$

$$\text{If} \quad M_{uy}^{-} > 0 \quad \text{then} \quad M_{uy}^{-} = 0 \quad \text{and} \quad M_{ux}^{-} = M_{xx} - \left|\frac{M_{xy}^2}{M_{yy}}\right|$$

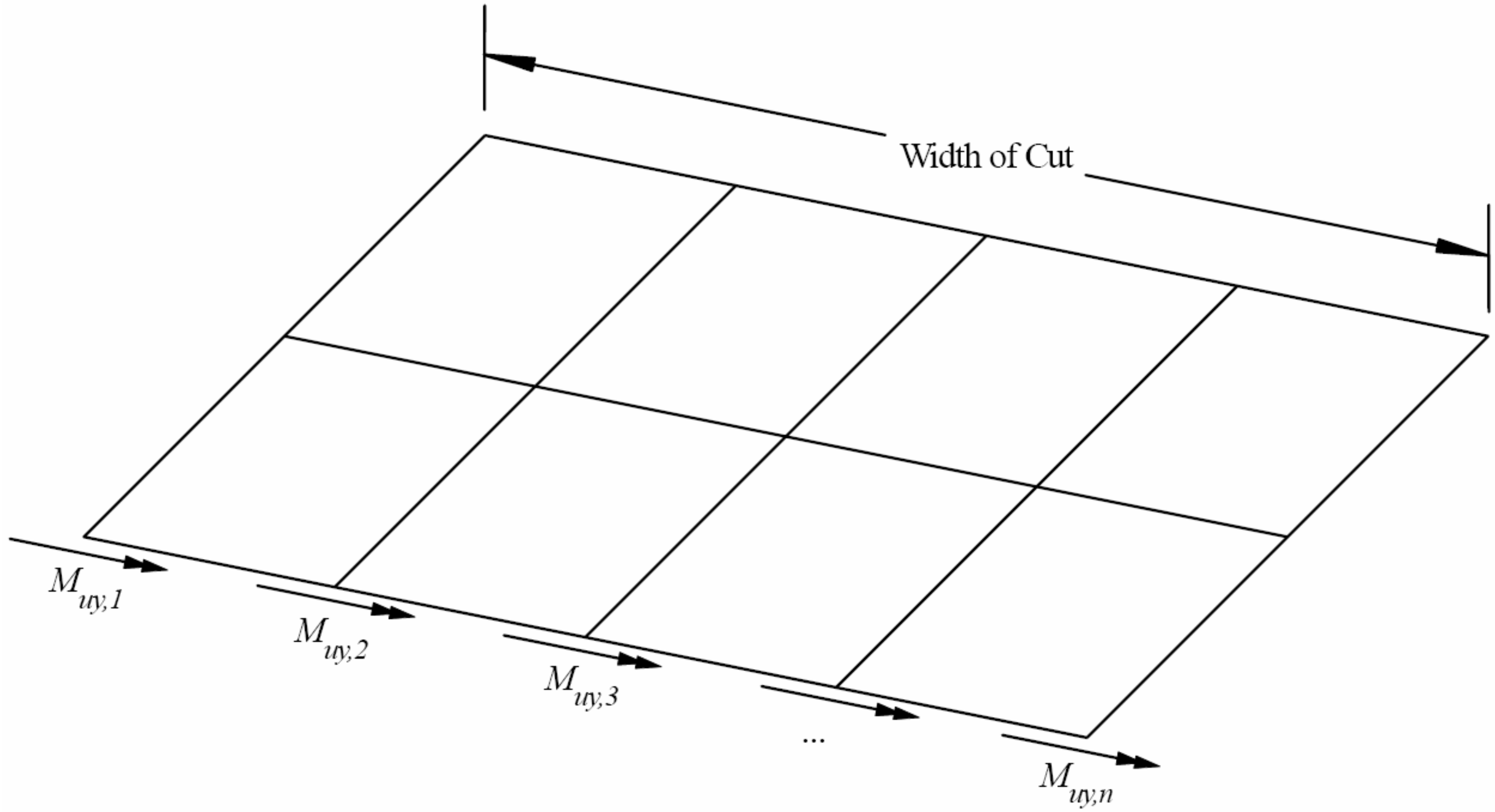
W&A - Design for Average Effect



$$M_B^+ = \sum_{i=1}^{n} \left(\frac{x_{i+1}^{cut} - x_{i-1}^{cut}}{2} \right) \times M_{uy,i}^+$$

$$M_B^- = \sum_{i=1}^{n} \left(\frac{x_{i+1}^{cut} - x_{i-1}^{cut}}{2} \right) \times M_{uy,i}^-$$

W&A - Design for Maximum Effect



$$M_B^+ = (x_n^{cut} - x_1^{cut}) \times max(M_{uy,i}^+ \text{ for } i = 1 \ldots n)$$
$$M_B^- = (x_n^{cut} - x_1^{cut}) \times max(|M_{uy,i}^-| \text{ for } i = 1 \ldots n)$$

Method 2 - Element Forces

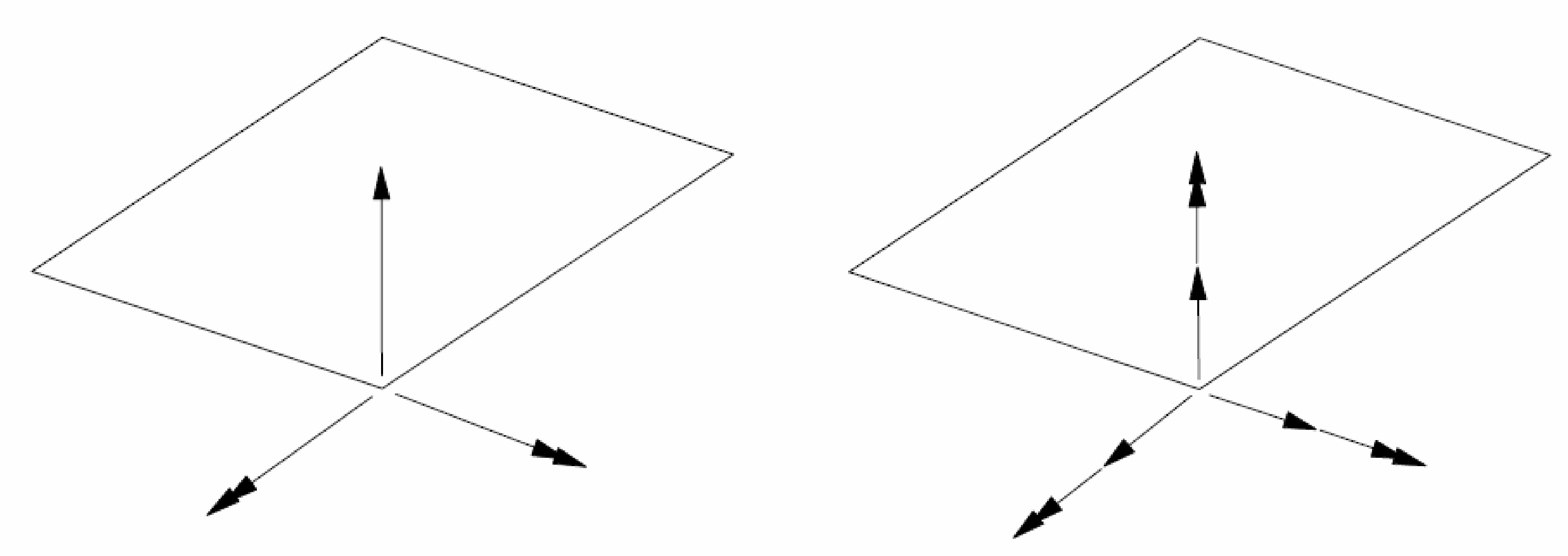
- Computed based on element equilibrium

$$k_e \cdot d_e - P_e$$

- Performs well because guaranteed to satisfy equilibrium
- Provide a force or moment corresponding to each DOF per element
- Can be thought of as “member end forces”

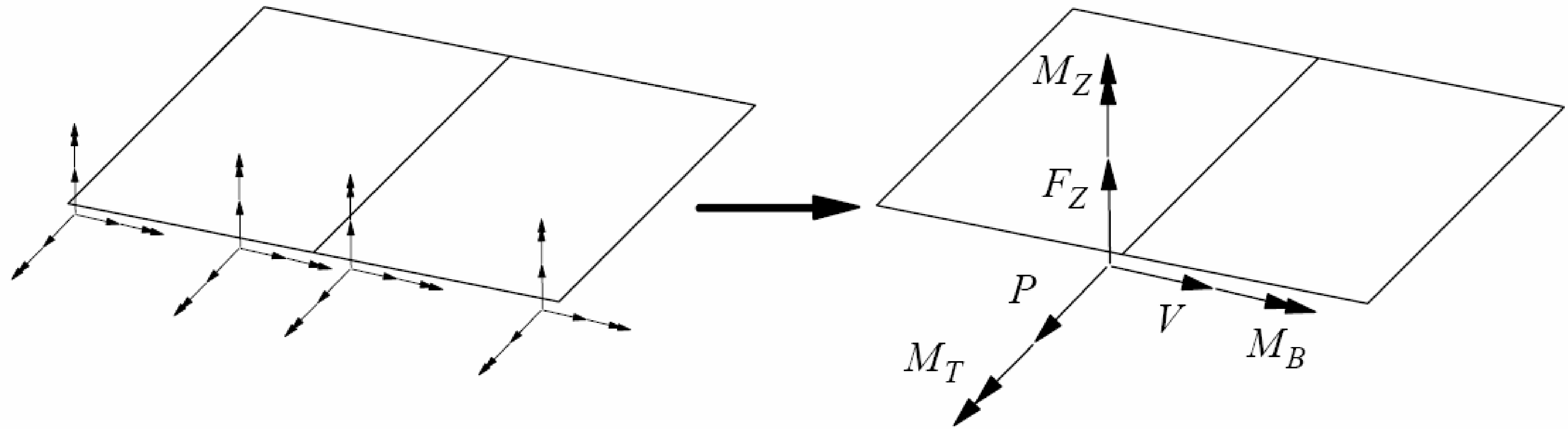
Element Forces at a Node

- 2 Moments, 1 Shear (Plate Bending)
- 3 Moments, 3 Forces (Shell Element)



Bending Moment - Element Forces

- Computed from equilibrium of entire cut:



$$M_B = \sum_{i=1}^{n} \sum_{k=1}^{m} M_{i,k}$$

n = # of nodes on cut
m = # of elements at each node

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Element Forces in GT STRUDL

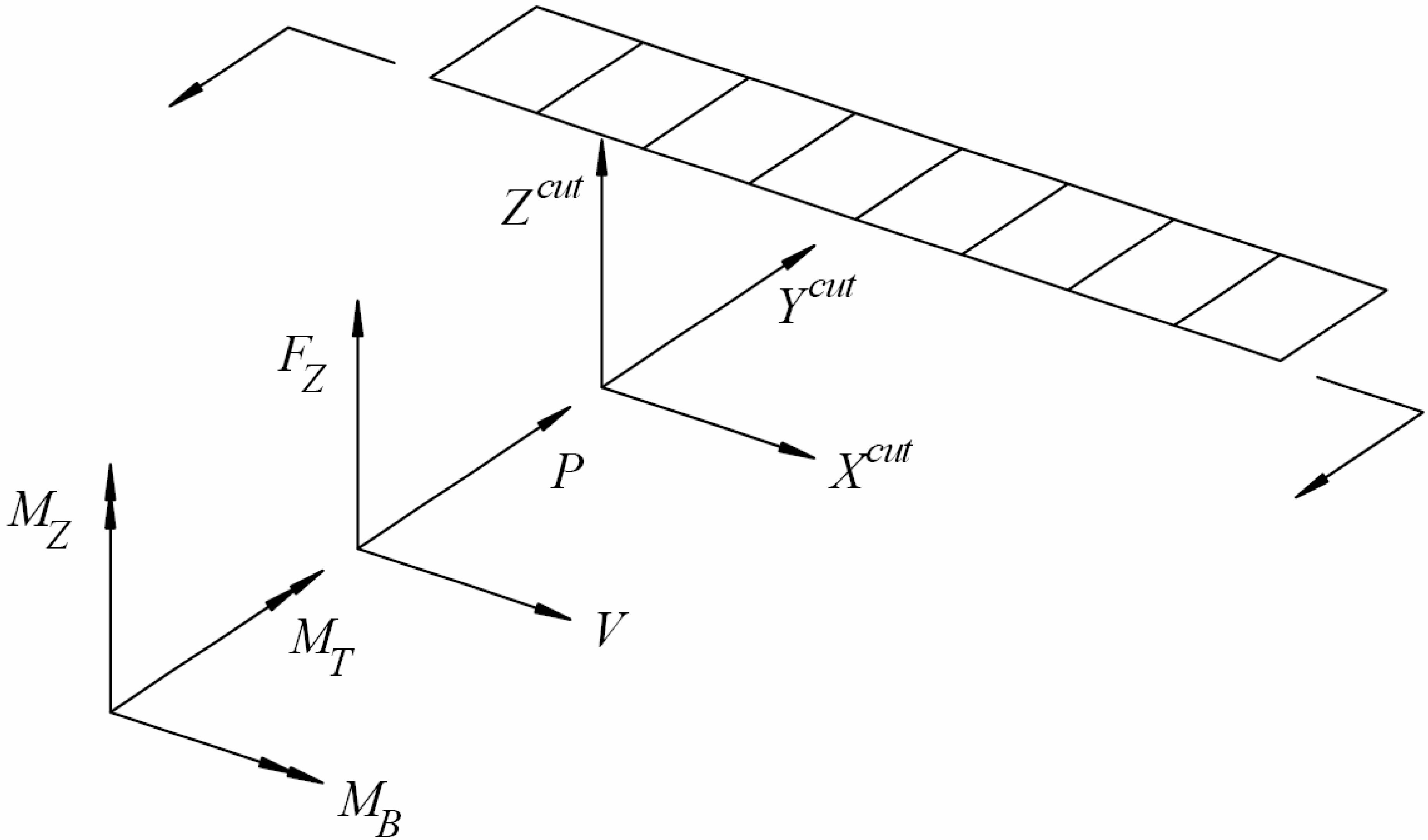
- CALCULATE RESULTANT Command
  - Determination of Design forces:
    - 3 Forces, 3 Bending moments in CUT C.S.
  - Input: List of Nodes & Elements along cut
  - Only list elements on ONE side
  - Syntax:

```
calculate resultant forces along cut i joints list elements list
```

Calculate Resultant Results

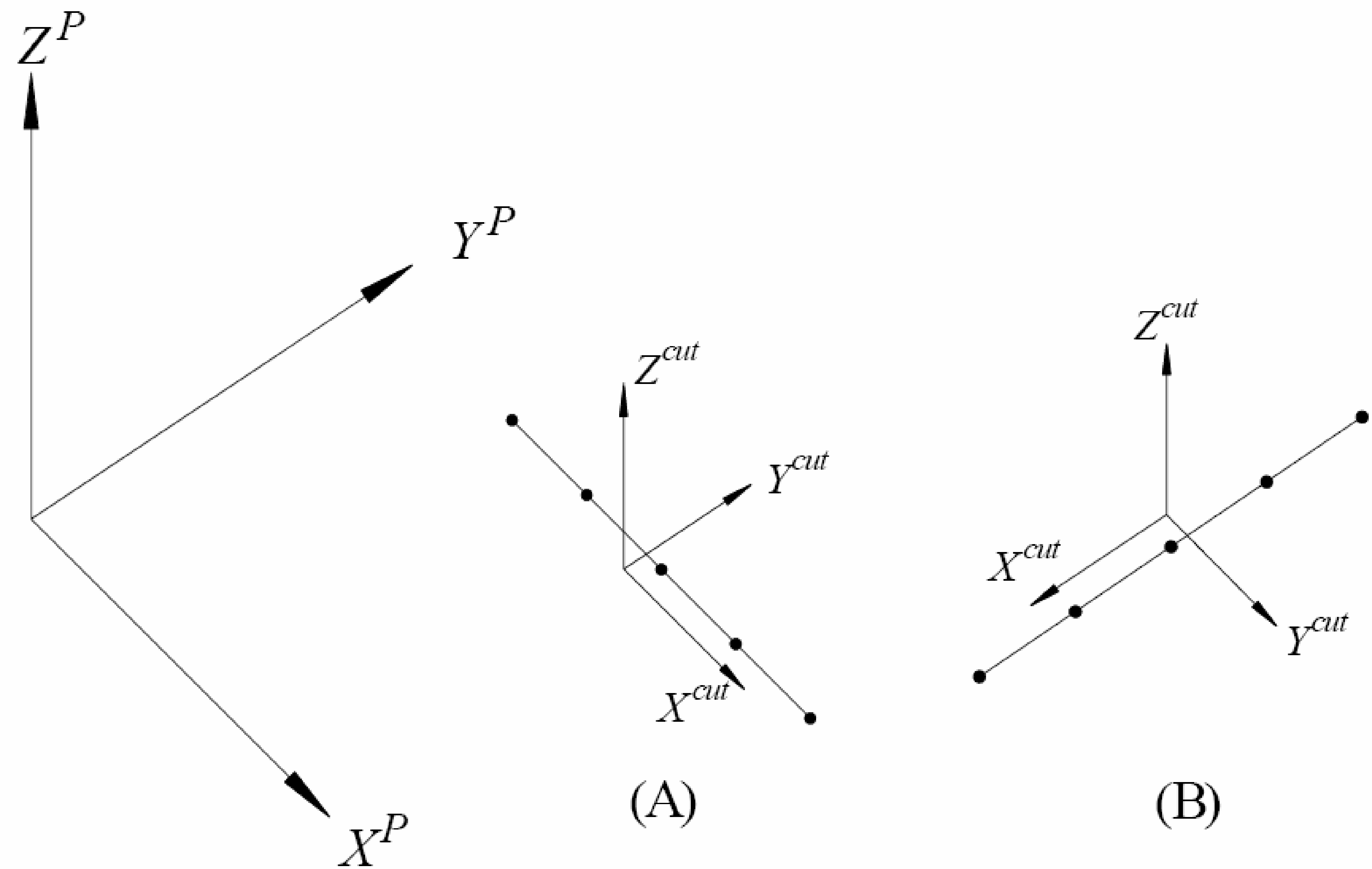
$$
\begin{aligned}
P &= \text{in-plane force normal to cut} \\
V &= \text{shear force parallel to cut} \\
F_Z &= \text{transverse (out-of-plane) shear force} \\
M_B &= \text{bending moment vector parallel to the cut} \\
M_T &= \text{torsional in-plane moment perpendicular to cut} \\
M_Z &= \text{rotational moment about out-of-plane axis}
\end{aligned}
$$

Calculate Resultant Sign Conv.



Cut Coordinate System

- (A) - General Case / (B) – Special Case



Command Implementation

- A slab design command was implemented based on both element forces and the Wood and Armer method
- Engineered to provide the engineer maximum possible control over the design of the system, including methodology, design parameters, and tolerances.

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DESIGN SLAB Command Syntax

$$
\underline{\text{DES}}\text{IGN } \underline{\text{SLAB}} \ (\underline{\text{REIN}}\text{FORCEMENT}) \ (\underline{\text{BY}}) \left\{ \begin{array}{l} \underline{\text{CALC}}\text{ULATE } (\underline{\text{RES}}\text{ULTANT}) \\ \underline{\text{WOOD}} \ (\underline{\text{AND}}\ ) \ (\underline{\text{ARM}}\text{ER}) \left(\begin{array}{l} \underline{\text{MAXI}}\text{MUM} \\ \rightarrow \underline{\text{AVE}}\text{RAGE} \end{array} \right) \end{array} \right\} -
$$

$$
(\underline{\text{AL}}\text{ONG}) \ (\underline{\text{CUT}} \left\{ \begin{array}{c} \text{`a'} \\ i_1 \end{array} \right\}) \left\{ \begin{array}{l} \underline{\text{JOI}}\text{NTS} \\ \underline{\text{NOD}}\text{ES} \end{array} \right\} list_1 \ \underline{\text{ELE}}\text{MENT} \ list_2 \ -
$$

$$
* \left\{ \begin{array}{l} \underline{\text{TOP}} \ (\underline{\text{FACE}}) \ (\underline{\text{BAR}}\text{S } i_2) \ (\underline{\text{SPA}}\text{CING } v_1) \\ \underline{\text{BOTTOM}} \ (\underline{\text{FACE}}) \ (\underline{\text{BAR}}\text{S } i_3) \ (\underline{\text{SPA}}\text{CING } v_2) \\ \underline{\text{BOTH}} \ (\underline{\text{FACE}}\text{S}) \ (\underline{\text{BAR}}\text{S } i_4) \ (\underline{\text{SPA}}\text{CING } v_3) \end{array} \right\} \left\{ \begin{array}{l} \underline{\text{OUT}}\text{ER } (\underline{\text{LAY}}\text{ER }) \\ \rightarrow \underline{\text{INN}}\text{ER } (\underline{\text{LAY}}\text{ER }) \end{array} \right\} -
$$

$$
(\underline{\text{COV}}\text{ER } v_4) \ (\underline{\text{TOL}}\text{ERANCE } v_5) \ (\underline{\text{TORS}}\text{IONAL } (\underline{\text{MOM}}\text{ENT}) \ (\underline{\text{WARN}}\text{ING}) \ v_6)
$$

Command Examples

- “DESIGN SLAB REINFORCEMENT ALONG CUT ‘A1’ BY CALCULATE RESULTANT JOINTS 1 2 ELEMENT 1 BOTH FACES BAR 5 SPACING 12.5 COVER 1.0”
- “DESIGN SLAB REINFORCEMENT BY WOOD AND ARMER NODES 1 2 ELEMENT 1 TOP FACE BAR 5 BOTTOM FACE SPACING 10 OUTER LAYER”

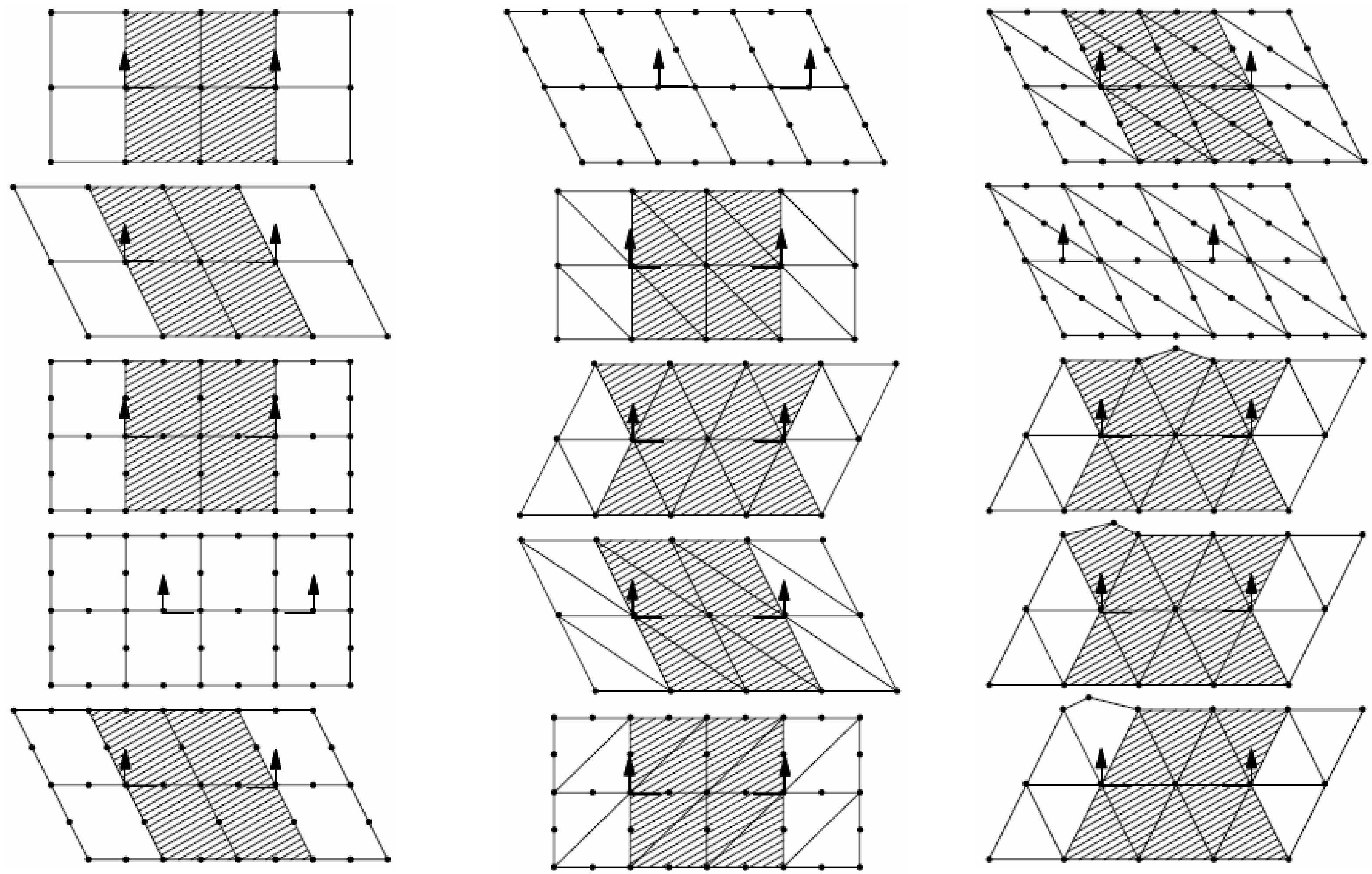
Building the Cut Definition

- User provides:
  - Start node of cut
  - End node of cut
  - Element in the plane of the cut
- Pre-processing algorithm then determines:
  - All interior nodes incident on the cut
  - Which elements contribute to the resultant force acting on the cut.

Element Contributions

- Appropriate element list critical for both element forces and Wood & Armer
- Primary concerns with element selection:
  - Preserving equilibrium
    - Adjacent cuts should not double-count end effects
  - Computing maximum effect
    - Free body of cut on both sides indicates that resultant moments computed on each side are not always equal, thus both sides must be checked for the maximum effect

Which Elements Contribute?

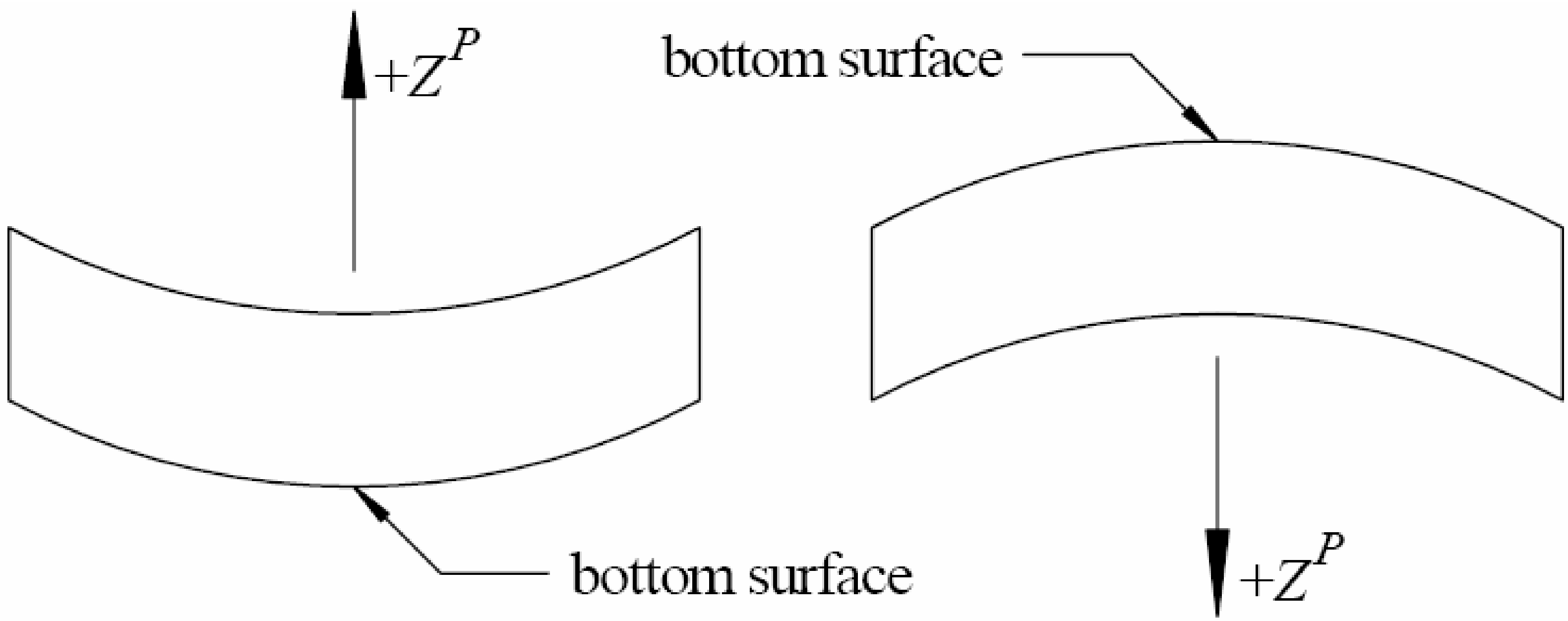


Design Algorithm

- At a given cut location, a user can:
  1. Design with no pre-constrained parameters
  2. Constrain bar size and design spacing
  3. Constrain spacing and design bar size
  4. Check a given bar size and spacing
- Design implemented in accordance with ACI 318-02 Building Code

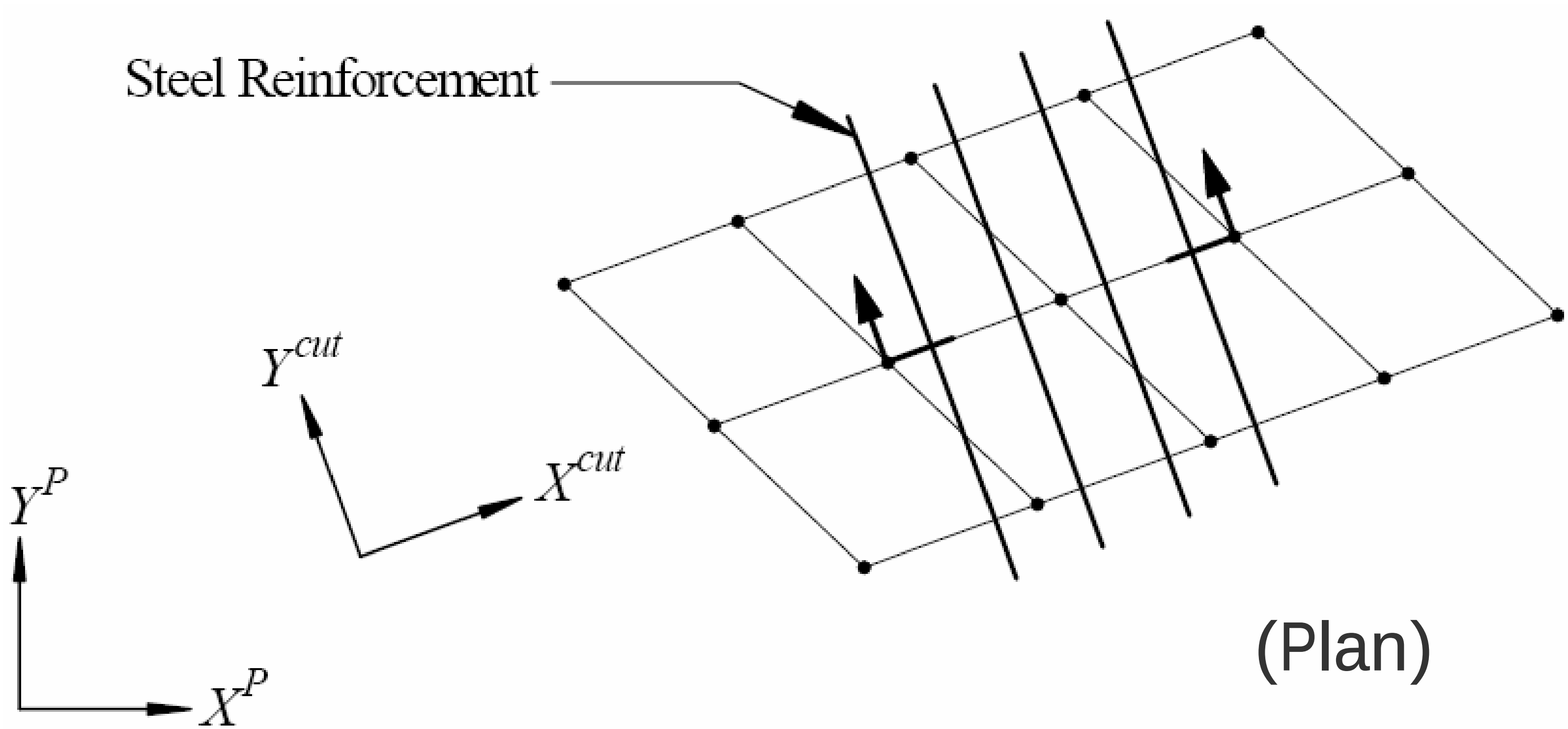
Interpretation of Results

- What does “top” or “bottom” mean?
- Depends on PLANAR AXES



Design Considerations

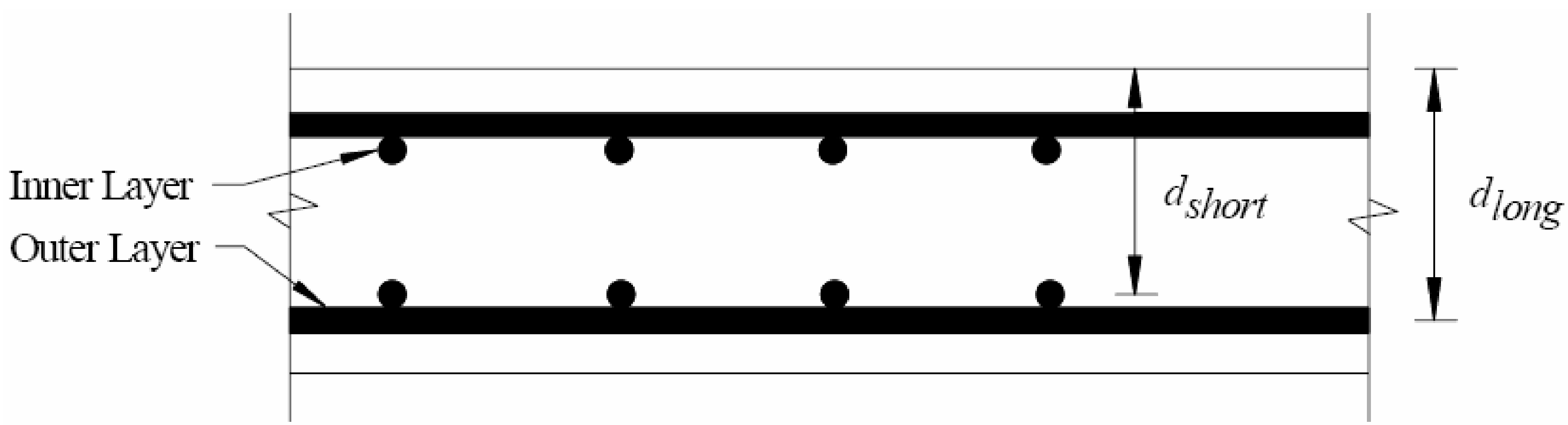
- Orientation of Reinforcement



(Plan)

Design Considerations, Cont'd

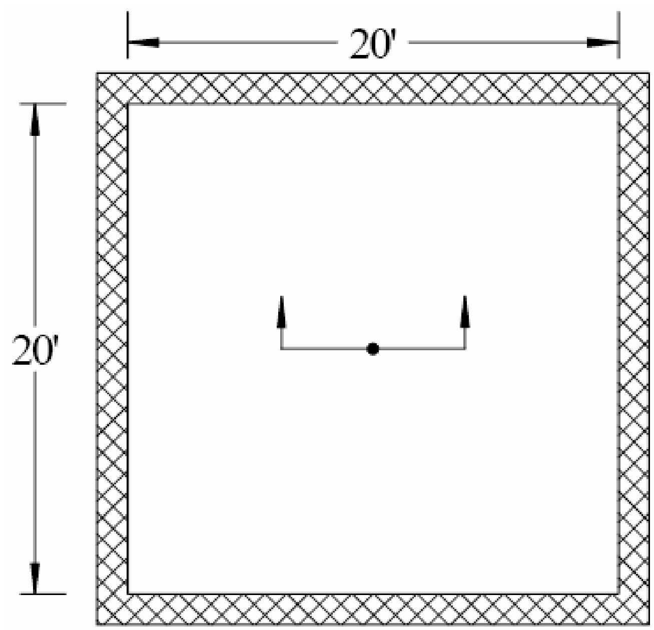
- Layer of reinforcement
- By default, design interior layer to be conservative

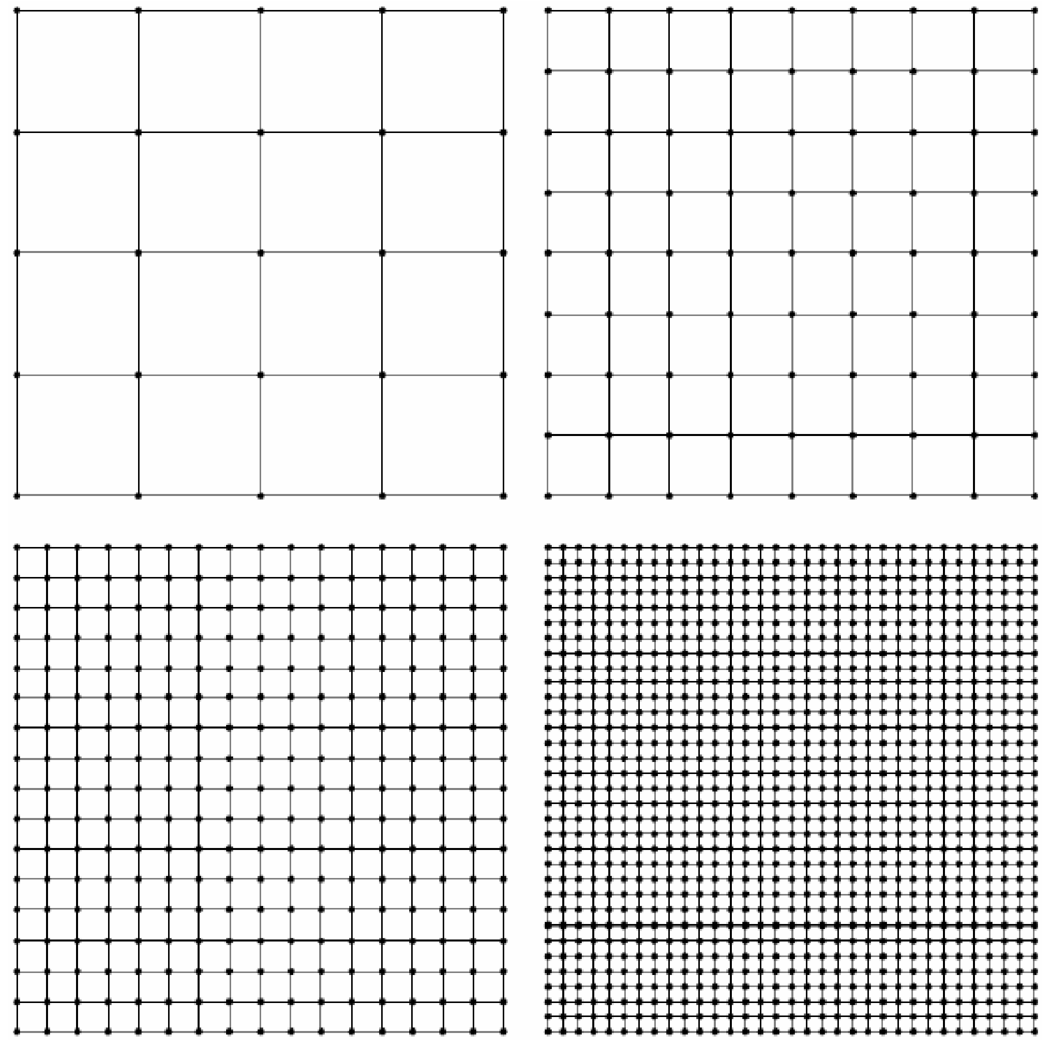


Case Studies

- Effect of convergence on design
- Validation with experimental data
- Comparison with ACI slab design methodologies
- Special Case – Strong torsion field

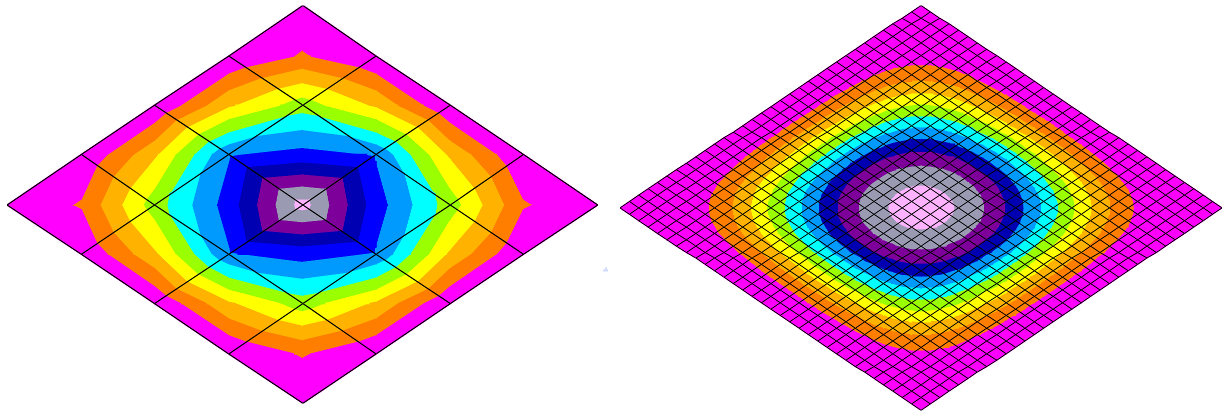
Example of Convergence



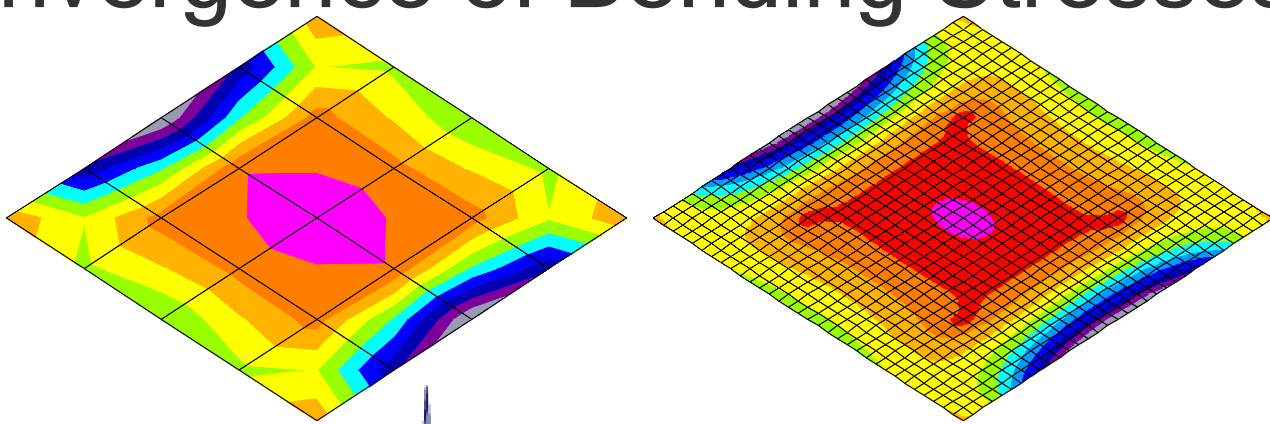


Contour Plot Comparison

- Convergence of Displacements

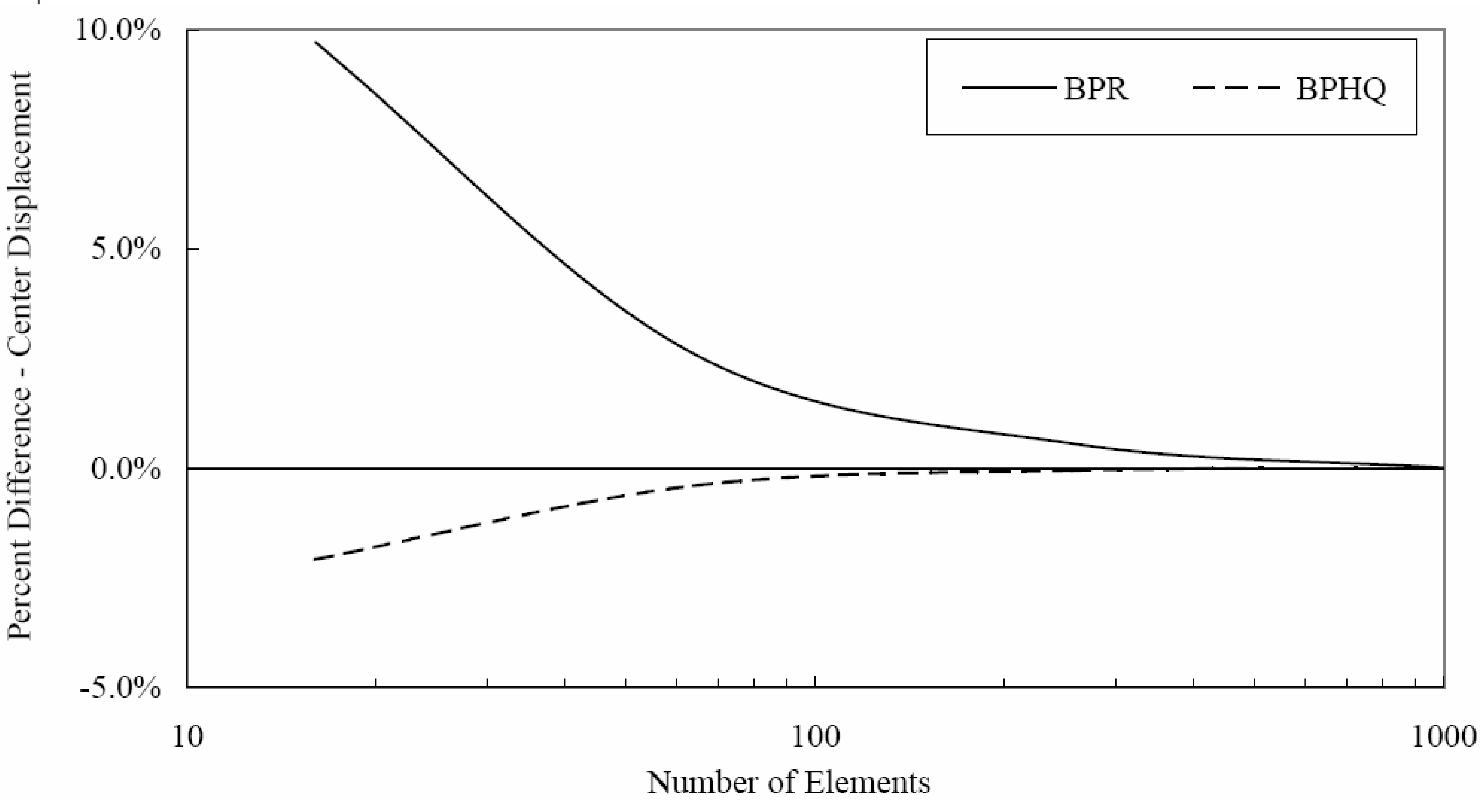


- Convergence of Bending Stresses



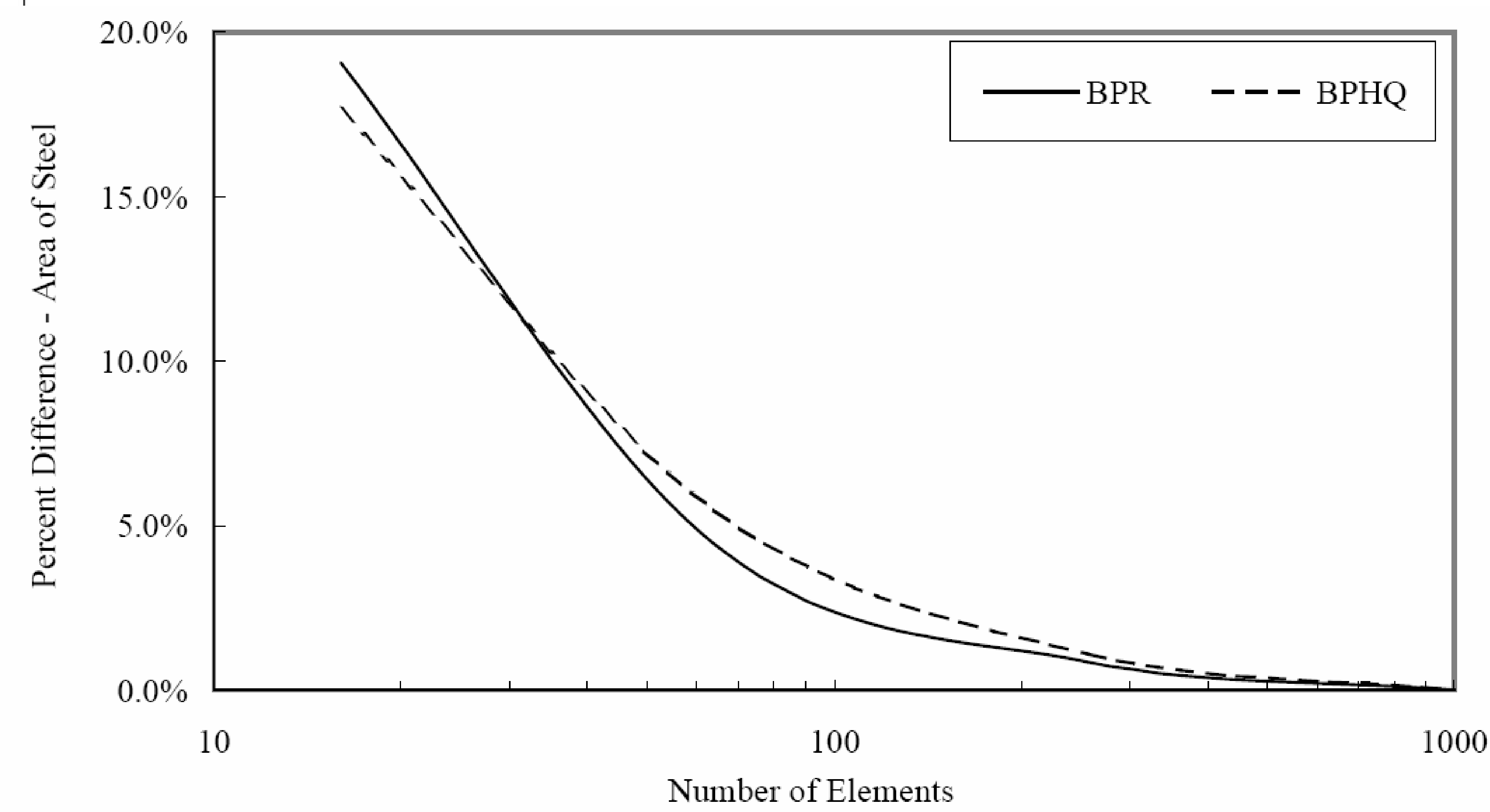
Convergence of Displacements

- Percent Change ~ 10 %



Convergence of Area of Steel

- Percent Change ~ 19 %



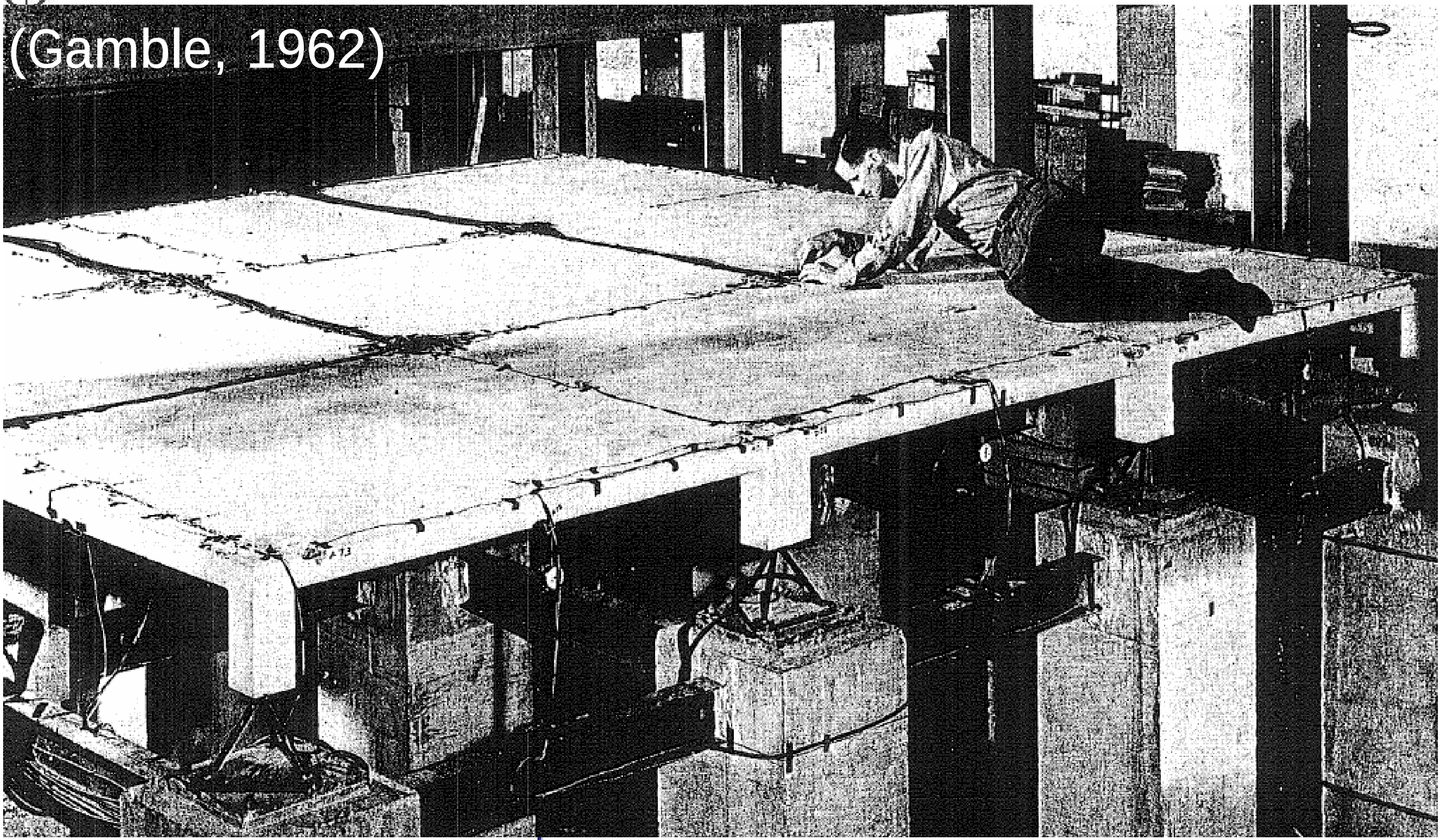
Convergence Conclusions

- Convergence must ALWAYS be evaluated
- Convergence of displacements DOES NOT guarantee convergence of design quantities such as stress, moment, area of steel
- Check convergence of the actual design quantity (in this case, area of steel)

Validation with Test Data

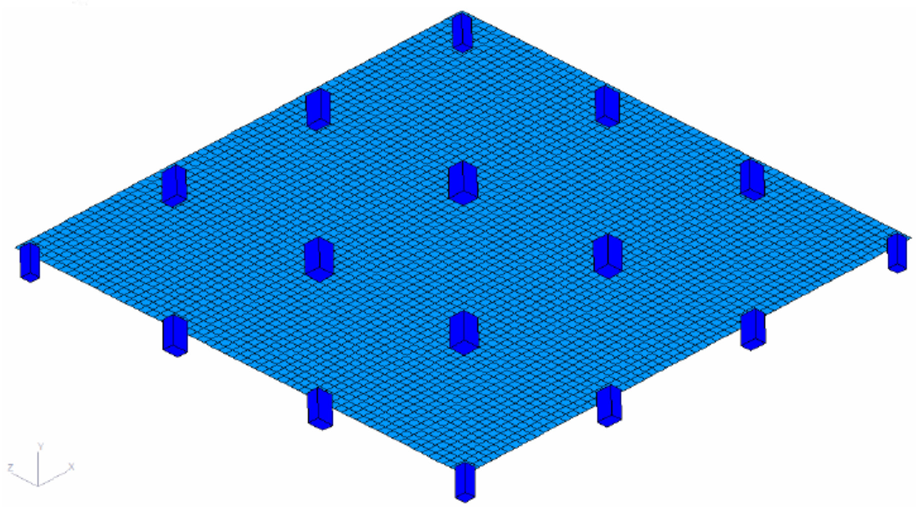
- Experiments carried out at University of Illinois by Dr. William Gamble (1962)
- If the finite element method doesn’t produce designs in agreement with real test specimens, it is not applicable for structural design.

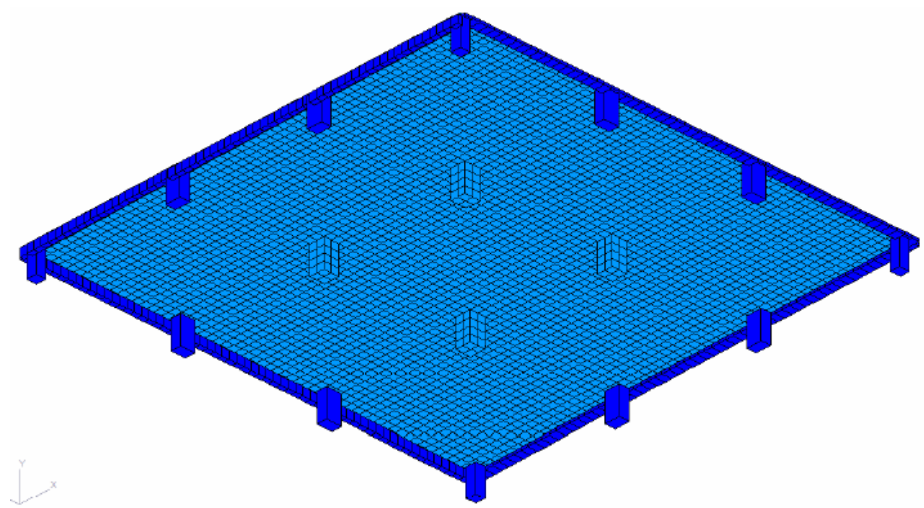
Picture of Test Structure



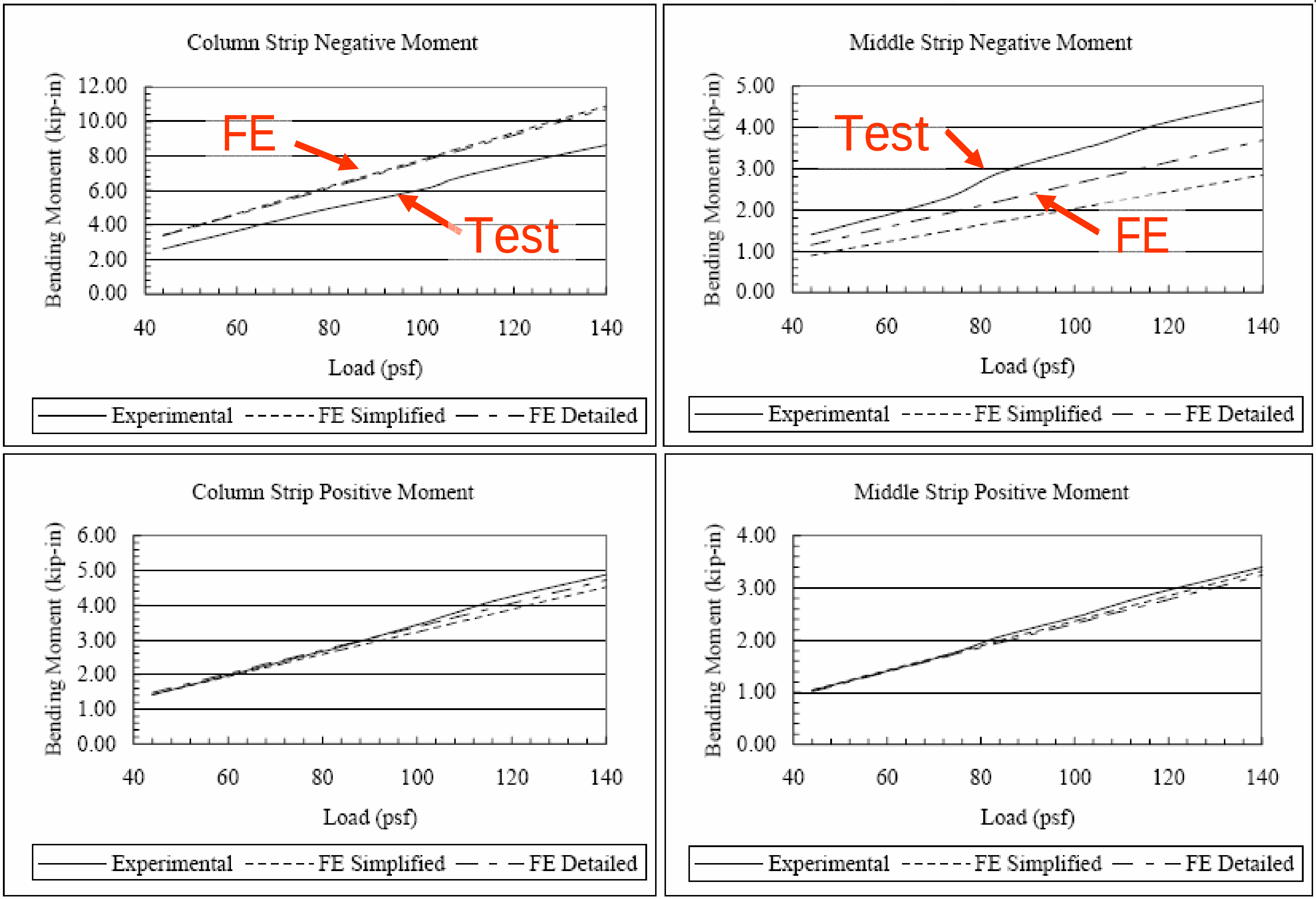
Model of Test Structure

- 2 Models
  - Edge beams neglected
  - Edge beams modeled and interior columns modeled as 3D solids



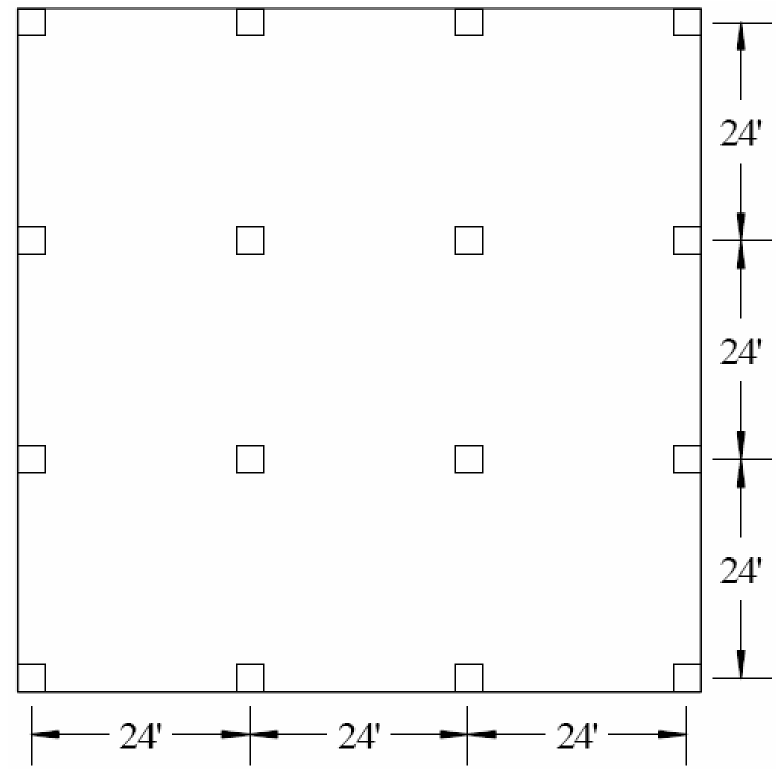


Model Results vs. Test Data



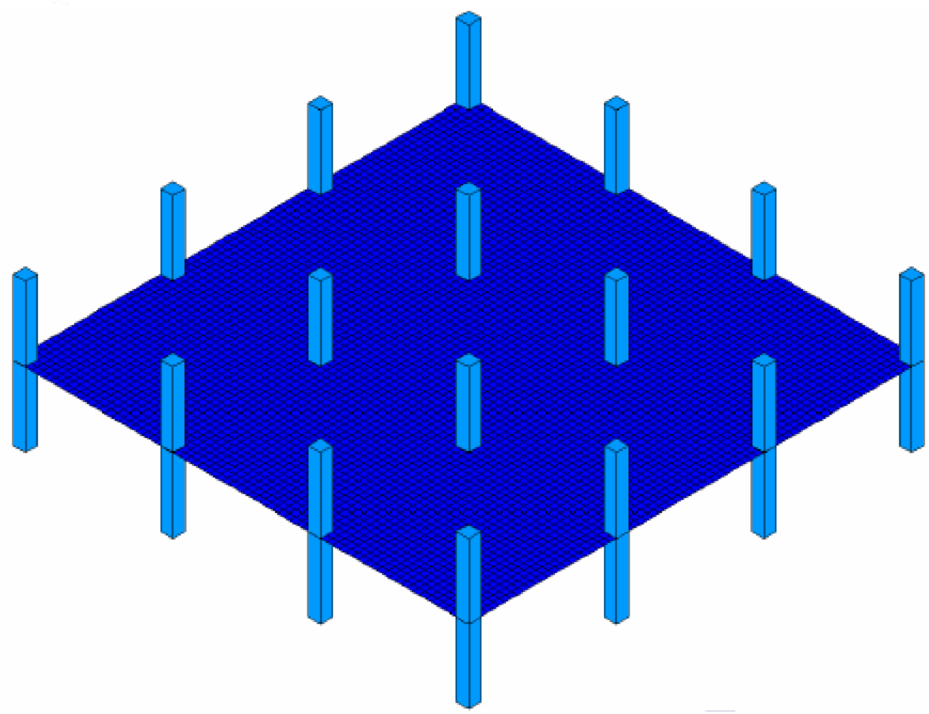
Example 2 - Geometry

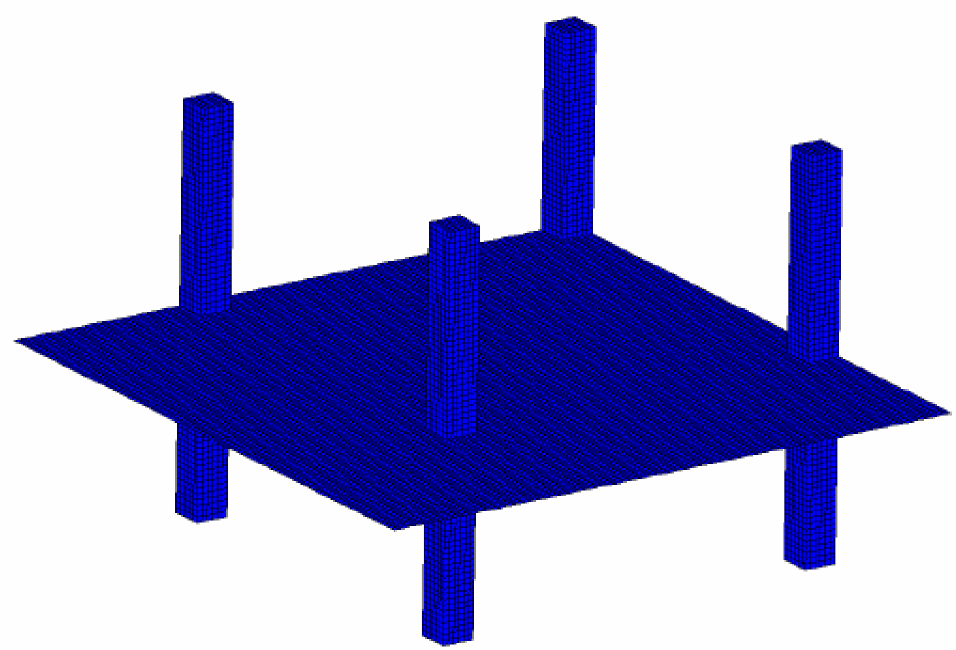
- Compare Finite Element Solutions with Typical Design methods
- Elevated flat plate
- Regular spacing
- Uniformly loaded



Example 2 - FEM Models

- Model 1 - Columns modeled as frame members with master-slave connections
- Model 2 - Columns modeled using 3D solid elements, ¼ modeled (symmetry)





Example 2 - Interior Results

Bending Moment Results (kip-ft):

Location	Direct Design	Equiv. Column	Wood & Armer (Rigid Solids)	DESIGN SLAB (Rigid Solids)	DESIGN SLAB (3D Solids)
Column Strip Negative Moment	-164.22	-180.53	-171.89	-174.95	-171.93
Column Strip Positive Moment	70.74	57.72	70.79	70.74	72.34
Middle Strip Negative Moment	-54.74	-60.18	-49.54	-48.67	-47.17
Middle Strip Positive Moment	47.16	38.48	51.29	51.23	51.97

Area of Steel Required (in^2):

Location	Direct Design	Equiv. Column	Wood & Armer (Rigid Solids)	DESIGN SLAB (Rigid Solids)	DESIGN SLAB (3D Solids)
Column Strip Negative Moment	6.15	6.81	6.21	6.58	6.48
Column Strip Positive Moment	2.55	2.07	2.55	2.55	2.61
Middle Strip Negative Moment	1.96	2.16	1.77	1.74	1.69
Middle Strip Positive Moment	1.69	1.37	1.84	1.84	1.86

Example 2 - Exterior Results

Bending Moment Results (kip-ft):

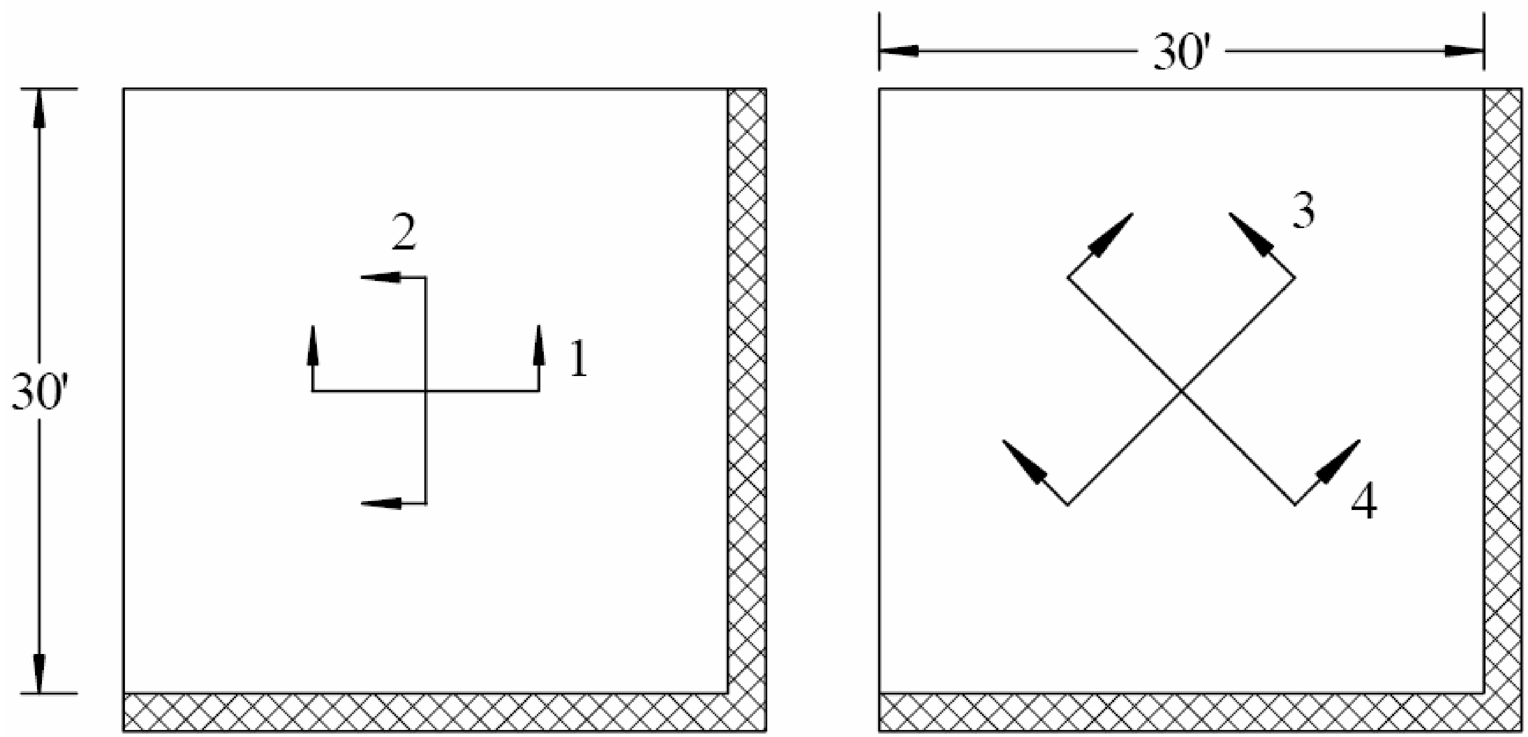
Location	Direct Design	Equiv. Column	Wood & Armer (Rigid Solids)	DESIGN SLAB (Rigid Solids)	DESIGN SLAB (3D Solids)
Column Strip Ext. Negative Moment	-87.58	-96.20	-149.12	-155.65	-150.96
Column Strip Positive Moment	105.10	96.18	81.07	81.83	84.58
Column Strip Int. Negative Moment	-176.85	-200.85	-173.34	-189.66	-186.80
Middle Strip Ext. Negative Moment	0.00	0.00	0.00	0.00	0.00
Middle Strip Positive Moment	70.07	64.12	62.57	63.21	65.04
Middle Strip Int. Negative Moment	-58.95	-66.95	-50.44	-46.66	-47.53

Area of Steel Required (in^2):

Location	Direct Design	Equiv. Column	Wood & Armer (Rigid Solids)	DESIGN SLAB (Rigid Solids)	DESIGN SLAB (3D Solids)
Column Strip Ext. Negative Moment	3.18	3.51	5.76	5.81	5.62
Column Strip Positive Moment	3.84	3.51	2.94	2.97	3.07
Column Strip Int. Negative Moment	6.66	7.64	6.66	7.18	7.06
Middle Strip Ext. Negative Moment	0.00	0.00	0.00	0.00	0.00
Middle Strip Positive Moment	2.53	2.31	2.25	2.28	2.34
Middle Strip Int. Negative Moment	2.12	2.41	1.81	1.67	1.70

Example 3 – Cantilever Slab

- Cantilever portion of continuous slab at corner region of building



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Example 3 - Cantilever Slab

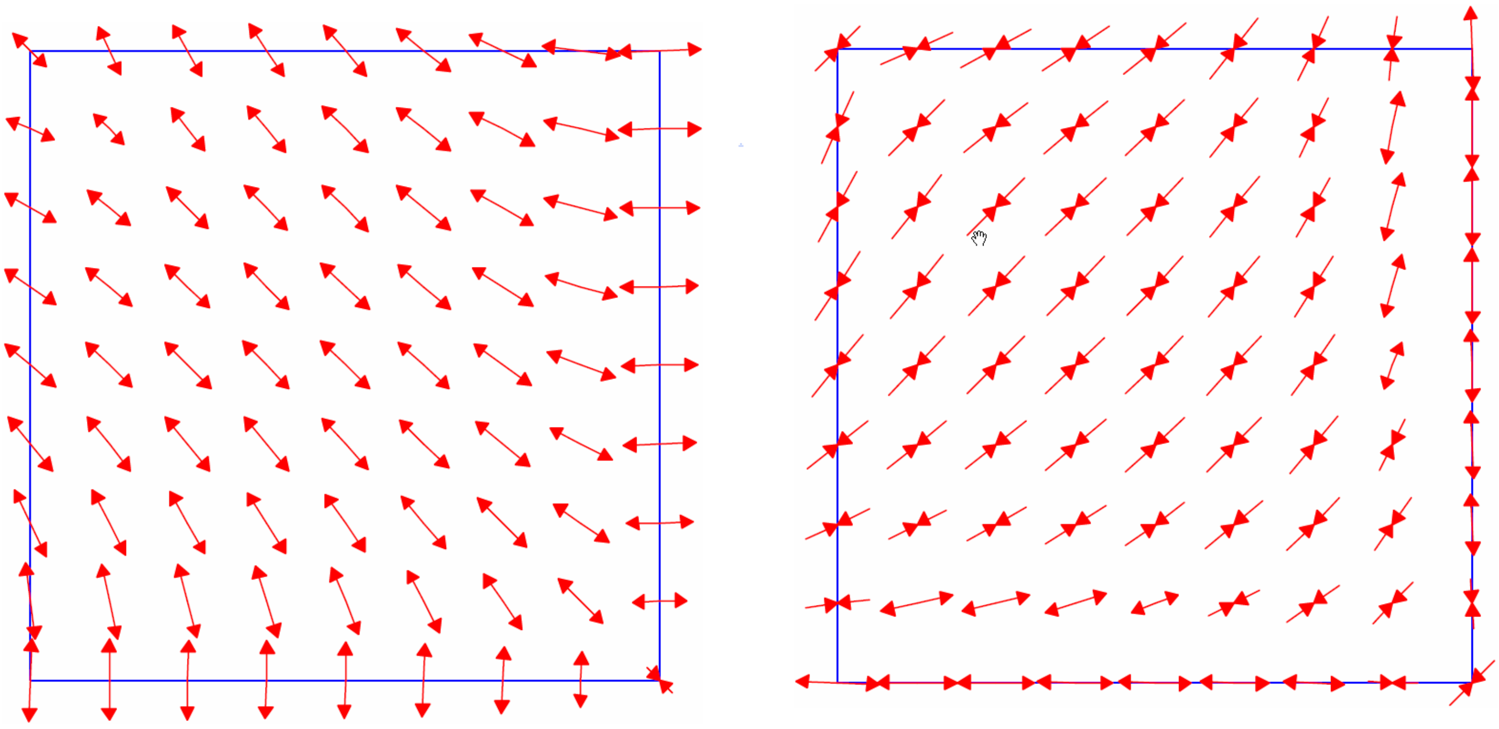
- Results of Element Force Method

Cut #	Bending Moment (k-ft)	Torsional Moment (k-ft)	$A_{s,required}$ (in^2)	Surface
1	6.51	240.09	0.1405	Bottom
2	6.51	-240.09	0.1405	Bottom
3	-176.10	0.00	3.8703	Top
4	172.85	5.87	3.7974	Bottom

- Cuts 1&2 from Wood and Armer:
  - Top: 3.85 in^2 / Bottom: 3.42 in^2

Example 3 – Principal Vectors

- Vectors showing directions of principal bending M1 and M2



Example 3 – Conclusions

- Element Forces should ONLY be used for cuts orthogonal with the principal bending directions
- Wood and Armer appropriately handles cuts oriented arbitrarily w.r.t. principal bending axes
- Failure to check could lead to seriously unconservative design!

Conclusions – Method Comparison

Method	Wood & Armer	Element Force
Advantage	Can be applied in strong torsion field	Always satisfies equilibrium
Disadvantage	Equilibrium not guaranteed. Must use fine mesh.	Does not represent strong torsion field

Final Conclusions

- Accurate modeling is fundamental.
- Finite elements tend to over-approximate the stiffness near columns and edges.
- Element forces can ONLY be used when cuts are orthogonal to directions of principal bending.
- Must understand behavior to determine whether to use element forces or moment resultants

Research Recommendations

- Incorporation of torsional moment in the element force methodology
- Validation with highly irregular structures, including geometric irregularities, openings, and concentrated effects
- Modeling of concrete nonlinearity (cracking, reinforcement bond-slip/plasticity)
- Design methodology for punching shear and moment transfer shear
- Validation of applicability under lateral loading

Recommended Program Enhancements

- Graphical selection of strips in GT MENU
- Automation of strip selection
- Incorporation of axial effects for shell elements to allow for extension to prestressed/post-tensioned systems

Questions?